Pseudo Differential Operators and Markov Process (Vol. III)

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This book is written about some kinds of Markov process (e.g. Feller processes , Lèvy processes and Hunt processes) and their relation with L^p-sub-Markovian semigroups or the martingale problem. It is also written about potential theory as a further topic.

I read this book from Chapter 3 ''Feller Processes'' and I'll go back to the privious parts if I need.

Chap. 3. Feller Processes

3.1 Projective Limits and Canonical Processes

♣ Corrections

p.47 l.10 $\pi_J := \pi_J^I$

p.49 l.23 $\mathcal{B}^K$-$\mathcal{B}^J$-measurable

♣ Comments

[The index set I]

I is a time parameter so that we can regard I as $[0, \infty)$ in many cases.

[Check that P_J in (3.9) is a prob. meas.]

[If the state space is $(\mathbb{R}, \mathcal{R})$ and $I = \mathbb{N}$, Kolmogorov's extension theorem implies Thm 3.1.2.]

[What the author means $X_J(P)$ in (3.18)]

I guess that he means $X_J(P) = P \circ X_J^{-1}$ so that P_J in (3.18) is the map from E^J to $[0, 1]$. Note that this P_J is different from P_J in (3.9).

♣ Examples

[Examples of polish spaces]

Polish spaces are so-called the completely separable metric spaces. As you know, ''norm spaces are metric spaces'' since if we define $d(x, y) = \|x - y\|$ for $x, y \in X$ (where X is a norm space), then $d(\cdot, \cdot)$ satisfies the axiom of metric. So I'll show some examples of separable Banach spaces.

1. $(\mathbb{R}^d, \|\cdot\|)$ where $\|x - y\| = (\sum_{1 \leqslant i \leqslant d} |x_i - y_i|^2)^{1/2}$ for $x, y \in \mathbb{R}^d$

(completeness)

(separabiilty)

$\mathbb{R}^d$ has the dense subset $\mathbb{Q}^d$. It is trivial that $\mathbb{Q}^d$ is countable.

2. $((l^p), \|\cdot\|)$ where $\|x\| = (\sum_{k\geqslant 1} \xi_k^p)^{1/p}$ for $x = (\xi_k)_{k\geqslant 1} \in (l^p)$

(completeness)

(separabiilty)

3. $(C[a,b], \|\cdot\|)$ where $\|f\| = \max_{a\leqslant x\leqslant b} |f(x)|$ for $f \in C[a,b]$

We can assume that $[a,b] = [0,1]$ since if we define g as $f(x) = g((x-a)/(b-a))$, then $g \in C[0,1]$ for $f \in C[a,b]$ and $f \leftrightarrow g$ in 1-1.

(completeness)

(separabiilty)

$C[0,1]$ has the dense subset $\{f; f(x) = \sum_{i\geqslant 0} q_i x^i, q_i \in \mathbb{Q}\}$. It is trivial that this subset is countable. This is proved by Weierstrass's approximation theorem.

4. $((L^p), \|\cdot\|_p)$ for $1 < p < \infty$

5. Abstract Wiener space with separable Banach space

Let (B, H, μ) be an abstract Wiener space i.e. B is a Banach space , $H(\subset B)$ is a Hilbert space which is dense in B and μ is a measure on B. If B is separable, then (B, H, μ) is a Polish space. Since this has a norm so that this is a metric space and B is a Banach so that B is complete.

[An example of canonical process]

I'll show a trivial example. Let $I = [0, \infty)$, $(\Omega, \mathcal{A}, P) = ([0,1), \mathfrak{B}[0,1), \text{Lebesgue meas.})$, $E = [0,1)$ and $\mathcal{B} = \mathfrak{B}[0,1)$.

Then $X_t(\omega) := \omega + t - [\omega + t]$ is a canonical process where $[\cdot]$ means the largest integer of $\cdot$.

3.2 Semigruops of Kernels, Transition Functions and Canonical Process

♣ Corrections

p.63 l.6 and l.7 ''I.4.13'' is I.4.1.3

p.65 l.12 running over

p.66 l.16 $<w', u_r>$ should be defined as follows,

$$<w', u_r> := \frac{1}{r} \int_0^r <w', T_s u> ds \quad \text{for } w' \in X^*$$

p.66 l.19 Krein-Milman theorem

p.68 l.14 $P_t : C_0(E) \to C_b(E)$

♣ Comments

[Lemma 3.2.15]

We define $\lim_{x\to\infty} k(x,C)=0$ as follows :

For all $\varepsilon>0$, there is a compact set K s.t. $|k(x,C)|<\varepsilon$ for $x\notin K$

[Theorem 3.2.18]

I guess that the author means ''xxx in weak topology'' by ''weak(-ly) xxx'' around here. For example, ''$\overline{\text{conv}}\,\{T_s u;\, s\in[0,r]\}$ is also weakly compact'' implies that $\overline{\text{conv}}\,\{T_s u; s\in[0,r]\}$ is also compact in weak topology in X. Since $X\subset X^{**}$ so that $\mathcal{O}_X\subset\mathcal{O}_{X^{**}}$.

♣ Details

3.3 A First Encounter with Sample Paths and Cadlag-Functions

♣ Corrections

p.76 l.22 $\lim_{t\to 0}\eta(t)=0$

p.77 l.5 (3.79)

$$\operatorname*{ess\,sup}_{t\geqslant 0}|\eta'(t)-1|\leqslant 1-e^{-\|\eta\|_L}\leqslant\|\eta\|_L$$

p.78 l.11 $d_D(\omega_1,\omega_3)\leqslant d_D(\omega_1,\omega_2)+d_D(\omega_2,\omega_3)$

p.79 l.18 i.e. the last line in (3.87)

$$\vee\left(\sup_{\eta_\nu(u)\wedge u\leqslant s\leqslant u}|\omega(\eta_\nu(u)\wedge u_\nu)-\omega(s)|\wedge 1\right)$$

p.79 l.23 $(\omega_\nu)_{\nu\in\mathbb{N}}$

♣ Comments

[Theorem 3.3.12]

What is ''locally uniform topology'' in this statement ?

♣ Details

3.4 Markov Processes and Feller Processes

♣ Corrections

p.91 l.12

$$\lim_{t\to 0}\int_t^\infty e^{-\alpha s}P_s u\,ds=\int_0^\infty e^{-\alpha s}P_s u\,ds$$

p.92 l.21 $(e^{-\alpha t}U_a u_\nu(X_t))_{t\geqslant 0}$

p.96 l.2 The left-hand side is :

$$\lim_{r \to 0} T_{t-r}(uT_r v)(x)$$

♣ Comments

♣ Details

3.5 The Shift Operator and the Strong Markov Property

♣ Corrections

p.100 l.3 $B \in \mathcal{B}^{(n)}$

p.101 l.16 -- 17 Then for h sufficiently small we find

p.105 l.8

$$P^{\mu}(Z'' \circ \theta_r - Z' \circ \theta_r > 0) = P^{\mu}(P^{X_r}(Z'' - Z' > 0)) = 0$$

p.106 l.1 We call $\dot{\sigma_A}$ the ...

p.107 l.2 A and σ_A ...

♣ Comments

♣ Details

3.6 The Martingale Problem for Feller Processes

♣ Corrections

p.109 l.15 Then cadlag modification ...

p.110 l.5 for $u(\in D(A))$ and $s \leqslant t$

p.113 l.22 (3.171) is :

$$P_1^{\mu}(Y_0 \in \Gamma) = P_2^{\mu}(Y_0 \in \Gamma) = \cdots$$

p.114 l.1 $0 \leqslant t_1 < t_2 < \cdots < t_m < t_{m+1}$

p.114 l.19 $\mu(x, dy)$

♣ Comments

♣ Details

3.7 Lèvy Processes and Translation Invariant Feller Semigroups

♣ Corrections

♣ Comments

♣ Details

3.8 A Summary of Some Path Properties of Lèvy Processes

♣ Corrections

♣ Comments

♣ Details

3.9 The Symbol of a Feller Process

♣ Corrections

♣ Comments

♣ Details

Chap. 4. The Martingale Problem

Chap. 5. L^p-sub-Markovian Semigroups and Hunt Processes